Rectification of light refraction in curved waveguide arrays

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An 'optical ratchet' for discretized light in photonic lattices, which enables to observe rectification of light refraction at any input beam conditions, is theoretically presented, and a possible experimental implementation based on periodically-curved zigzag waveguide arrays is proposed.

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Transport of discretized light in photonic lattices has attracted a great attention in recent years from both fundamental and applied viewpoints [1, 2]. Engineered photonic lattices offer the possibility of tailoring diffraction [3], refraction [4], and of shaping polychromatic light beams [5–7]. They also provide a rich laboratory system to mimic the dynamics of quantum particles driven or disordered periodic lattices, with the observation of the optical analogues of Bloch oscillations [1, 8], Anderson localization [9, 10] and dynamic localization [11–13] to name a few. Recently, the transport of ac-driven quantum particles in periodic lattices has received a great interest also for the possibility to achieve Hamiltonian ratchet effects, i.e. a rectified transport in absence of a net bias force and dissipation. Different ratchet models based on optical lattices, which use ultracold atoms as ideal systems, have been proposed [14–19], and detailed studies have investigated the relation between rectification and broken space-time symmetries [15, 17], the effects of atomic interactions [19], and the possibility to transport quantum information [16, 18]. In optics, the idea of realizing an 'optical ratchet' has received few attention to date, and limited to solitary waves in nonlinear systems [20–22]. In particular, in [20] a setup to observe ratchet effects for cavity solitons has been proposed using coupled waveguide optical resonators with an adiabatically-shaken holding pump beam, whereas in [21] control of soliton dragging in dynamic optical lattices has been investigated.

In this Letter a simple ratchet scheme is proposed in a *linear* optical system corresponding to rectification of discretized light refraction in an array of periodically-curved zigzag waveguides, which may be realized with currently available femtosecond laser writing technology [23]. We consider two interleaved arrays of single-mode channel waveguides, a primary array A_n and an auxiliary array B_n, in the zigzag geometry shown in Fig.1(a), which is similar to the one recently realized in Ref. [23] to investigate second-order coupling effects. As opposed to Ref. [23], the propagation constant of modes in waveguides B_n is assumed to be detuned by a relatively large amount σ from the propagation constant of modes in waveguides A_n, a condition that can be achieved in practice by e.g. lowering the writing velocity of the auxiliary array. The optical axis of the waveguides, which lies in the (X, Z) plane, is assumed to be periodically-curved along the paraxial propagation direction Z with a bending profile $x_0(Z)$ of period $2z_0$. In the waveguide reference frame $z = Z$ and $x = X - x_0(Z)$, where the arrays appear to be straight, under scalar and paraxial assumptions light propagation at wavelength λ is described by the following Schrödinger-like equation (see, for instance, [11])

$$i\hbar\frac{\partial\psi}{\partial z} = -\frac{\hbar^2}{2n_s}\nabla^2\psi + [n_s - n(x, y)]\psi + n_s\ddot{x}_0(z)x\psi \quad (1)$$

where $\hbar = \lambda/(2\pi)$ is the reduced wavelength, n_s is the refractive index of the substrate medium, $n(x, y)$ is the refractive index profile of the (straight) arrayed structure, ∇^2 is the transverse Laplacian, and the dot denotes the derivative with respect to z. As discussed e.g. in [11], light propagation in the optical system mimics the temporal dynamics of a quantum particle in a binding potential $n_s - n(x, y)$ driven by an external (inertial) force $-n_s\ddot{x}_0(z)$ with zero mean. In the tight-binding approximation, light transfer among the waveguides may be described by a set of coupled mode equations for the amplitudes a_n and b_n of modes trapped in waveguides A_n and B_n, respectively, which may be cast in the following form (see, for instance, [24])

$$\begin{aligned} i\dot{a}_n &= -\kappa_{aa}\left[a_{n+1}\exp(-2i\Theta) + a_{n-1}\exp(2i\Theta)\right] + \quad (2)\\ &- \kappa_{ab}\left[b_n\exp(i\sigma z - i\Theta) + b_{n-1}\exp(i\sigma z + i\Theta)\right], \\ i\dot{b}_n &= -\kappa_{bb}\left[b_{n+1}\exp(-2i\Theta) + b_{n-1}\exp(2i\Theta)\right] + \quad (3)\\ &- \kappa_{ab}\left[a_n\exp(-i\sigma z + i\Theta) + a_{n+1}\exp(-i\sigma z - i\Theta)\right], \end{aligned}$$

In Eqs.(2) and (3), κ_{aa} and κ_{bb} are the coupling rates between adjacent waveguides in the primary and auxiliary arrays, respectively, κ_{ab} is the cross-coupling rate [see Fig.1(a)], $\Theta(z) = \pi n_s\dot{x}_0(z)d/\lambda$, d is the spatial period of the arrays, and σ is the propagation constant mismatch between waveguides in the two arrays.

For straight arrays and large mismatch $|\sigma| \gg \kappa_{ab}, \kappa_{aa}, \kappa_{bb}$, light dynamics in the two arrays is decoupled and a broad beam, which excites the primary array A_n at an incidence angle θ smaller than the Bragg

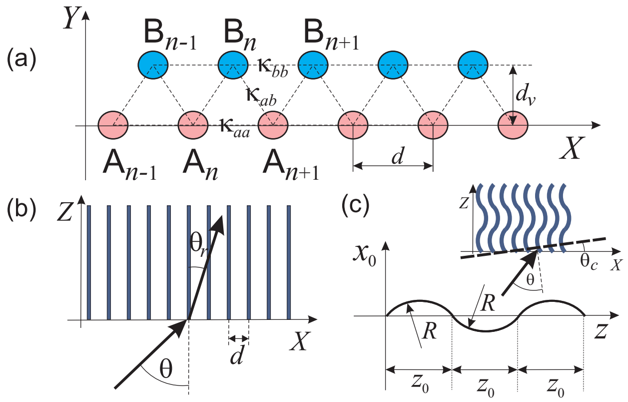

Fig. 1. (Color online) (a) Schematic of a zigzag waveguide array, composed by a primary array A_n and an auxiliary array B_n. (b) Light refraction in a straight waveguide array. (c) Axis bending profile $x_0(z)$ for the observation of light refraction rectification. The inset in (c) shows a schematic of the periodically-curved array.

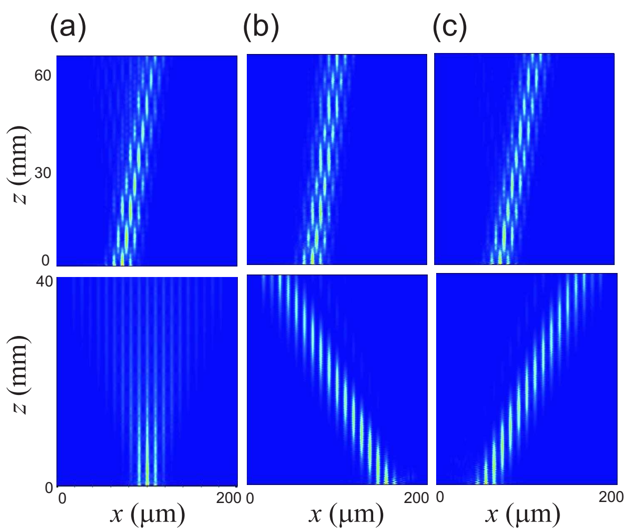

Fig. 2. (Color online) Rectified refraction (pseudocolor map of integrated intensity distribution) in a 65-mm-long periodically-curved zigzag waveguide array (upper images), and corresponding behavior expected for a straight zigzag array (bottom images), for an input elliptical beam exciting a few waveguides in the primary array at different tilting angles: (a) $\theta = 0$, (b) $\theta = -0.5\theta_B$, and (c) $\theta = 0.5\theta_B$. The values of other parameters are: $\lambda = 633$ nm, $n_s = 1.45$, $\Delta n_A = 0.003$, $\Delta n_B = 0.0032$, $w = 3$ μm, and $d = 9$ μm.

angle $\theta_B = \lambda/(2d)$ [see Fig.1(b)], will diffract and refract at an angle $\theta_r(\theta) = 2\kappa_{aa} d \sin(\pi\theta/\theta_B)$ as in an ordinary tight-binding linear lattice [4]. Rectification of beam refraction, i.e. a net transverse drift of discretized beams averaged over all possible incidence angles θ, is obviously absent because $\theta_r(-\theta) = -\theta_r(\theta)$. Even for a single array A_n, curved to mimic a dc, ac or combined dc-ac driving fields, a net transport of wave packets -averaged over different initial conditions- is generally unlikely, as discussed in Ref. [14]. To achieve a ratchet effect, we introduce the auxiliary lattice B_n and assume that both arrays are periodically-curved with a bending profile $x_0(z)$ made of a sequence of circular arcs of (paraxial) length z_0 and constant curvature with alternating sign $\ddot{x}_0(z) = \pm 1/R$, as depicted in Fig.1(c). Such a bending profile corresponds to an ac square-wave force $-n_s \ddot{x}_0(z)$, and to a triangular shape for the function $\Theta(z)$ entering in Eqs.(2) and (3). Note that, since in this case the waveguide axis at the entrance plane $z = 0$ is tilted by an angle $\dot{x}_0(0) = z_0/(2R)$, the incidence angle θ should be now measured with respect to a plane tilted by an angle $\theta_c = n_s z_0/(2R)$ from the entrance facet [11], as shown in the inset of Fig.1(c). To achieve rectification of light refraction in the primary array over one undulation cycle, the radius of curvature R and spatial period z_0 of alternations are designed to satisfy the conditions $\dot{\Theta}(z) = \pm\sigma$ and $\kappa_{ab} z_0 = \pi/2$, i.e.

$$R = \pi n_s d/(\lambda|\sigma|), \quad z_0 = \pi/(2\kappa_{ab}). \tag{4}$$

In the first semi-cycle of undulation, where $\dot{\Theta}(z) = \sigma$, neglecting nonresonant terms Eqs.(2) and (3) reduce to $i\dot{a}_n = -\kappa_{eff} b_n$ and $i\dot{b}_n = -\kappa^*_{eff} a_n$, where $\kappa_{eff} = \kappa_{ab}\exp(i\phi)$ and $\phi = \sigma z - \Theta$ is a constant phase shift. This means that waveguide A_n turns out to be coupled solely with waveguide B_n, and light transfer between them occurs like in an ordinary synchronous directional coupler. For a propagation length z_0 equal to one coupling length $z_0 = \pi/(2\kappa_{ab})$, light trapped in waveguide A_n is thus fully transferred into waveguide B_n. At the successive propagation semi-cycle, where $\dot{\Theta}(z) = -\sigma$, Eqs.(2) and (3) yield $i\dot{a}_{n+1} = -\kappa_{eff} b_n$ and $i\dot{b}_n = -\kappa^*_{eff} a_{n+1}$ with $\kappa_{eff} = \kappa_{ab}\exp(i\phi)$ and $\phi = \sigma z + \Theta$, i.e. waveguide B_n turns out to be coupled solely with waveguide A_{n+1}. For a propagation length z_0 equal to one coupling length, light trapped in waveguide B_n is thus fully transferred into waveguide A_{n+1}. Therefore, after a full modulation cycle, *an arbitrary* light distribution in the primary array A_n is shifted, without distortion, by one unit from the left to the right. The result is a net drift of refracted light, with a locked refraction angle $\theta_r = d/(2z_0)$ *independent of the initial incidence angle θ and beam shape*, and a suppression of discrete diffraction. Similarly, an arbitrary light distribution in the secondary array B_n is shifted by one unit but this time from the right to the left. As the ratchet effect results from an alternating synchronization of couples of waveguides in the array ($A_n \to B_n$ and $B_n \to A_{n+1}$), different bending profiles $x_0(z)$, such as sequences of sinuosoidally-curved waveguides with alternating amplitudes mimicking the driving field of Ref. [16], do not generally generate a ratchet effect because of incomplete decoupling of the transport dynamics. As compared to other ratchet schemes in tight-binding lattices [14,16,18], our zigzag configuration offers the advantage of spatially separating a primary lattice (in which we want to realize a net transport) from an auxiliary one, which is transiently excited.

The above analytical results are strictly valid within

a tight-binding model of Eq.(1) and in the averaging limit, which neglects weak light transfer among waveguides due to non-resonant coupling terms. We checked the possibility of observing rectification of light refraction beyond such approximations by a beam propagation analysis of Eq.(1). In the simulations, circular channel waveguides with a super-Gaussian index profile $\Delta n(x,y) = \Delta n_{A,B} \exp\{-[(x^2+y^2)/w^2]^3\}$, with radius w and index changes Δn_A and Δn_B in the primary and auxiliary arrays, are assumed. The vertical distance d_v between the two arrays is chosen to be $d_v = \sqrt{3}d/2$, i.e. the zigzag is defined by an equilateral triangle. As an input beam, we typically assumed an elliptical Gaussian beam, with a tilting angle θ (in the x direction) smaller than the Bragg angle θ_B to excite the lowest band of the array. The vertical beam position and size are adjusted to couple a few waveguides of the primary array. Examples of rectified light refraction, observed in the curved arrays for a few values of beam incidence angle θ, are shown in Fig.2. In the figure, the integrated beam intensity distribution $\int |\psi(x,y)|^2 dy$ versus propagation distance z is shown and compared to the behavior that one would observe if the arrays were straight, i.e. with usual discrete light refraction patterns. For the chosen array parameters and wavelength, the coupling rates and propagation mismatch are estimated to be $\kappa_{aa} = \kappa_{bb} \simeq \kappa_{ab} \simeq 1.939$ cm^{-1} and $\sigma \simeq 16.19$ cm^{-1}, corresponding to a radius of curvature $R = 4$ cm and a semi-cycle period $z_0 = 8.1$ mm according to Eq.(4). The numerical results clearly show that, for curved arrays, light refraction always occurs at the angle $\theta_r = d/(2z_0)$, regardless of the beam incidence angle θ, and discrete diffraction is well suppressed as a result of the controlled transport mechanism. Of course beam refraction is reversed when the position of the elliptical input beam is shifted to excite waveguides of the auxiliary array. As an example, Fig.3 shows rectification of light refraction when the array B_n is excited at the input by an elliptical Gaussian beam at normal incidence. Note that, as compared to Fig.2(a), the refraction angle of discretized light is now reversed.

In conclusion, rectification of light refraction in a periodic photonic structure, which provides an example of an Hamiltonian ratchet system, has been proposed, together with a possible experimental implementation based on periodically-curved zigzag waveguide arrays.

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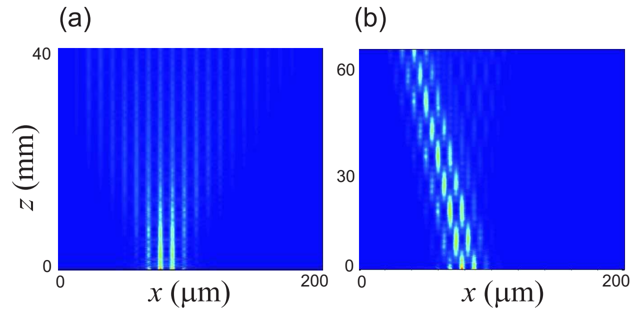

Fig. 3. (Color online) Same as Fig.2(a), but for an elliptical Guassian input beam vertically displaced to excite waveguides of the auxiliary array. In (a) the arrays are straight, whereas in (b) they are periodically-curved.

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